DOCUMENTO CEDES/47

THREE-GAP MODELS, OPTIMAL GROWTH AND THE ECONOMIC DYNAMICS OF HIGHLY INDEBTED COUNTRIES

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Buenos Aires
CEDES
1990

1. Introduction.

A major issue of discussion in current Latin-American economics is a variety of the so-called "three-gap" models. These models extended the "two-gap" models developed during the late sixties by adding a "fiscal/financial" gap to the traditional "saving" and "external" gaps.

The inclusion of the new gap into the analysis was the consequence of two stylized facts that characterized the Latin-American development process in the eighties.

The first important fact was that after the debt crisis the current account disequilibrium of indebted countries became structural rather than transitory. The second one was that the external crisis also took the form of a fiscal crack since the governments of these countries held a proportion of nearly a hundred percent of their external debts. The emergence of recent hyperinflationary episodes in several Latin-American countries had certainly to do with the public sector solvency problems and the impossibility of these governments to simultaneously honour their domestic and external debts.

Therefore, the burden of the adjustment fell on the public sector. More specifically, government investments were dramatically reduced and because of the existence of a "crowding-in" effect a maximum rate of growth for the whole economy was determined.

The "three-gap" literature elaborated on these effects yielding further insight into both the working of the financial system in underdeveloped countries and the role of the public sector budget constraint in determining macro-equilibrium and the growth rate of the economy.[1]

However, the problem has not been presented within the optimal growth theory context and hence lacks an expression for optimal long-run capital stock. Conversely, optimal growth theory attributes a negligible interest to instantaneous constraints and most models under discussion nowadays by macroeconomists elaborate on budgetary conditions to be fulfilled on average -the so-called isoperimetric constraints. The "third gap" belongs to the first set of constraints rather than to the second one.

The importance of the integration of both perspectives for the design of economic policy in highly indebted economies is herein emphasized.

The plan of the paper is as follows. The next section is devoted to discussing the emergence of the "third gap" its consequences on the growth process and the present situation of highly indebted countries. Section 3 is addressed to present the basic three-gap model in an optimal growth framework; the impact of the "fiscal-gap" on production through a reduction in the provision of public goods or collective factors of production is also discussed there. In section 4 the solution of a simple linear problem is analyzed when both the external and the fiscal gaps are binding. Section 5 deals with the possibility that these gaps prevent the economy from reaching the long-run capital stock; it is also shown there that a hysteresis phenomenon could arise. Section

[1] Basic references for two-gap models are Chenery and Bruno (1962) and Chenery (1961). On three-gap models see Fanelli et.al (1987), Bacha (1989) and Taylor (forthcoming).

6 summarizes the main conclusions of the paper.

2. The emergence of the "third gap" and the debt crisis.

The development of the research program in the "gaps" tradition has been closely related to the real problems that appeared in the Latin-American development process during the postwar period. In fact, the two-gap model was developed in order to explain the most relevant stylized facts that were observed during the import-substitution-led growth processes previous to the debt crisis (which took place at the beginning of the present decade).

The most important stylized fact that the two-gap approach was intended to explain was the recurrence of the balance-of-payments crises during the growth process, followed by a period characterized by an excess of savings over investment and, therefore, of relatively high unemployment.

The main diagnosis of this approach was that the difficulties that the semi-industrialized economies of the region faced in achieving macro equilibrium (i.e. difficulties faced in closing both gaps at the same time) were due to two main reasons. The first one was that the import-substitution process had contributed to the build up of an economic structure characterized by a chronic scarcity of foreign currency in a context of rationing of external credit for developing countries. The second reason was that macroeconomic variables such as imports, exports and savings showed low responsiveness to changes in relative prices (specially in the short-run) and hence, the macroeconomic adjustments following balance-of-payments crises worked through income rather than substitution effects.

The emergence of any sizeable deficit in the current account (following, for example, a negative term-of-trade shock) could not be financed. Given the price inelasticity of imports and exports, the only way to close the external gap was to induce a recession in order to get the required amount of foreign exchange via the negative impact on imports of a falling domestic income.

The introduction of a third gap for analyzing issues related to the financial structure of the economy and the role of the government budget constraint was also encouraged by the changes that took place in the Latin-American economic structure.

These changes were generated by the debt crisis in the early eighties which prompted an interruption in the process of continuous growth experienced during the post-war period. Likewise, the magnitude of the disequilibrium caused by the debt crisis in the balance of payments was so huge that it marked the disappearance of growth and the beginning of a period of deep macroeconomic instability. This macroeconomic instability included in almost all cases a government financial crack, capital flight and, in some countries like Argentina, Brazil and Bolivia, hyperinflationary episodes.

The first stylized fact that the three-gap models take into account is that even though the external constraint is still a tight one with regard to growth, its character has changed since the debt crisis.

In fact, before the debt crisis, the emergence of periodic balance-of-payment imbalances played a fundamental role in inducing short run macroinstability but in the long run it did not impede growth. The scarcity of foreign currency took the form of periodic disalignments between the flow of domestic absorption and the flow of domestic income but, at the cost of generating a certain degree of instability in the evolution of the economy, the economy was able to attain a high growth rate in the long run.

After the debt crisis, on the contrary, the current account disequilibrium has become a structural one. That is, it is not a short-run phenomenon.

This change in the character of the external gap is due to the fact that the present current account disequilibrium does not stem from a disadjustment between the flows of income and absorption but from the stock-flow disequilibrium between the stock of the external debt and the flow of income of the economy. This imbalance is reflected in a huge debt/national income ratio. In other words, while the scarcity of foreign exchange before the eighties created a liquidity squeeze in the short run, during the present decade the persistence of the current account deficit has been the reflex of a solvency (rather than liquidity) problem. As a consequence, the scarcity of foreign exchange is now generating negative growth in the long run rather than recession in the short run.

The second stylized fact is that the external crisis also took the form of a fiscal one.

This was a direct consequence of the fact that in Latin America, typically, the government held a proportion of nearly a hundred percent of the country's external debt. In this way, the country's solvency problems were transferred to the public sector. Therefore, the impact effect of the crisis was to increase the ratio between the stock of the debt and total government revenues to unsustainable levels.

In order to reduce their financial exposure the governments in the region reacted by significantly lowering public expenditures and in fact, the burden of adjustment fell on public investment and therefore on growth.

It was also observed that the falling government investment induced a kind of negative "crowding-in" effect on private investment and in this way, the government budget constraint became an independent constraint on the growth rate. That is, a third gap appeared related to the government budget constraint.

The third stylized fact has to do with the financial side of the economy.

As was said, an important consequence of the external squeeze during the eighties was macroeconomic instability. This led to a process of deep and rapid demonetization of the economy because of both the acceleration of inflation and the increment of the financial fragility not only in the public but also in the private sector. The natural consecuence of the demonetization of the economy was the "dollarization" of the financial relationships associated with a capital flight process.

Due to these new developments on the financial side, some authors have formalized the third gap as an inconsistency between the composition of both the supply and the demand of financial assets that cannot be eliminated by movements of the relevant yields of financial assets[2]. That is, the shift of the portfolio composition of the private sector in favor of foreign assets has generated an excess demand for external financial assets and, necessarily, an excess supply of domestic ones. In this context, even though there were an overall equality between investment and saving (i.e. even though the domestic gap were closed) there would be a disequilibrium on the financial side of the economy. Since the main source of the supply of domestic assets is the government, ceteris paribus, the "dollarization" of the economy would render it impossible to finance a given public deficit.

In other words, the denationalization of domestic savings has put a tight financial constraint on the public sector leading to a

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[2] On this subject see Fanelli and Frenkel (1989) and [illegible]lli (1988).

higher than needed fiscal adjustment (given the domestic agents' propensity to save). In this way, the financial squeeze that the public sector is undergoing appears as a strong constraint on growth justifying the introduction of a third gap in the two-gap model.

Even though the explanatory power of the three-gap models has proved to be strong, it has often been criticized as being based on ad-hoc premises. In this way, this model has suffered an attack similiar to that received by the IS-LM model: it is useful but not based on first (and therefore sounder (?)) principles. One of the objectives of this paper is to show that even in an explicit intertemporal optimization framework, the main results established by the three-gap approach are maintained provided that both all the relevant constraints faced by the economic agents and some key structural features of the economies under study are correctly specified.

To be more specific the rigidities stemming from the lack of price-responsiveness of some key macroeconomic variables must be present in the model.

Moreover, in the present context of rationing in both the domestic and external credit markets, the isoperimetric conditions that usually appear in an optimal growth program must be replaced by instantaneous constraint to be fulfilled at each point in time[3]. This means that the economic agents -and particularly the public sector- might not be able to take advantage of the smoothing properties of credit on consumption.

In other words, even though a necessary condition for solvency is that of being able to pay the debt in the long run, the existence of rationing in the credit market means that the debtor

[3] This is the kind of assumption that modern literature abandons; instead the analysis of interior solutions is emphasized -see Blanchard and Fischer (1989), p.49.

should be able to meet further constraints with regard to the point-in-time structure of its liabilities.

However, the main objective of the paper is not to discuss the relationship between the gap and the neoclassic traditions but the more substantive one of trying to obtain results by means of a cross-fertilization of the gap approach with that of the optimal growth theory. This kind of exercise is very fruitful since the intertemporal aspects of the gap approach have not yet been systematically studied.

Specifically, the model to be developed in the following sections sheds some light on the following problems: the "hysteresis effect" of the present economic crisis and the stabilization policies that were followed during the eighties; the relationship between growth and the strategies oriented to the maximization of external payments which are implicit in some policy recommendations, and the critical role of the public sector in fostering growth given the importance of this sector in providing basic infrastructure for private production.

3. The gap problem in an optimal growth framework.

This section is addressed to present the characteristics of the production and consumption processes of our model and the constraints that the existence of the afore-mentioned gaps impose on the economy.

Let's begin with production and consumption. Let k stand for the capital/labor relation and let

$$y = \Gamma(k)\ F(g\epsilon) \tag{1}$$

represent the economy's production function, where g corresponds to government expenditures per capita in real terms (excluding interest payments) and y stands for GDP. While it is assumed that $\Gamma(k)$ has the properties of a well-behaved neoclassic production function, it is also assumed that total production depends non-negatively on g: $\epsilon F'(\epsilon g) \geq 0$.

That is, we are assuming that collective factors of production or intermediate inputs exist. We are including in the definition of the economy's structure the (often ignored) fact that private production is not possible -specially in semi-industrialized economies- if the government does not provide the needed infrastructure for economic activity such as roads, harbors and so on. Notice, however, that considering the constant ϵ to be zero would give the traditional results of independence between g and production[4].

With regard to consumption we shall maintain that the programming bureau seeks to maximize:

$$(2) \qquad \int_{o}^{T} u(c)e^{-\delta t}\, dt,$$

where c is per capita consumption in real terms and δ is the rate of discount of future utilities. Likewise, we will assume later that $u(c)=u_o c$ with u_o being a positive constant, i.e., u is a linear function of consumption.

Regarding the constraints stemming from the balance sheets of the agents in an intertemporal setting, if the international credit markets were open to the country it would be natural to impose the following condition on the balance of payments:

[4] Bacha (1989) presents a formal three-gap model that assumes complementarity between public and private investment. That hypothesis is not necessary here.

$$\int_0^T \phi \, e^{-rt} \, dt = 0,$$

where ϕ corresponds to the net flow of foreign savings to the country. Under this specification ϕ might take any value at each point in time provided that "on average" the debt were repaid. For example it could be positive for a period allowing domestic absorption to exceed domestic income.

When international credit markets are rationed, however, this condition must be replaced by:

$$M - X \leq - f$$

where M and X stand for real imports and exports per capita respectively and $(-f)$ corresponds to net international flows of capital into the country (net of foreign interest payments)[5]. f is a constant exogenously given (provided that the rationing scheme is not manipulable in the sense of Benassy (1982)).

As was already mentioned, the existence of these kinds of "imperfections" on the financial side of the economy, that take the form of credit rationing in both domestic and external credit markets, have led to the development of the three gap approach. Let's state the budget restrictions of the agents in order to show how rationing gives rise to gaps.

From the balance sheet identity of the private sector, it follows that the private surplus must equal the flow demand for domestic assets (net of interest payments on such assets):

(3) $$(y - c - I - A) = L$$

[5] Traditional models of Chenery-Bruno (1962) and Chenery (1961) should be mentioned here regarding this constraint.

where I stands for gross investment and A for government revenues. The government budget constraint is:

$$(4) \qquad (A-g) = -L + f$$

That is, the government surplus must be allocated either to reduce domestic liabilities (-L) or to increase the demand for foreign assets, f (notice that both variables are measured net of interest payments).

The balance of payments (or the rest of the world budget constraint) is given by:

$$(5) \qquad (M - X) = - f.$$

If the rest of the world runs a surplus, there will be a fall in their liabilities.

In order to introduce into the model the rigidities in the price responsiveness of imports and exports we will assume that exports are a proportion of GDP[6]:

$$(6) \qquad X = xy$$

and that the production process requires a proportion m_0 of imported intermediate inputs while some fraction m_1 of total investment corresponds to imported capital goods, i.e.:

$$(7) \qquad M = m_0\, y + m_1\, I$$

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[6] To justify this hypothesis it is often argued that international markets are closed to the products of the country. Bacha (1984) argues that in the cost-of-adjustment version of the model such a relation assumes a state of excess supply in world product markets in the short run (the so called structuralist view) but that in the long run exports expand following domestic capacity.

We are now ready to present the gaps. Assume as has already been mentioned that international credit markets are rationed (i.e., f is exogenous) and that the economy is undergoing a "dollarization" process that implies that the incremental demand for domestic assets is zero (i.e. L=0). Then, from (3) and (4) we can deduce the fiscal gap:

$$(8) \qquad (g-A) \leq -f.$$

If we assume that the variable A is not under the control of the government because of tax evasion, tax erosion or the Olivera-Tanzi effect[7], (i.e. $A=\bar{A}$), then[8]:

$$g \leq \bar{A} - f.$$

Given the assumed form of the production function, the fiscal/financial gap exerts a severe constraint on growth.

Taking into account (6) and (7) we obtain the external gap from (5):

$$(9) \qquad [(x-m_0)/m_1]\ y - f/m_1 \geq I.$$

The availability of external credit (net of interest payments) imposes a ceiling on the economy's growth possibilities. Furthermore, if we take into account that:

$$(10) \qquad I = \sigma k + \dot{k}$$

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[7] The analysis of the effects of lags in collection of taxes on the dynamics of inflation and government financing was originally developed by Olivera (1967). An application to the Argentine case was conducted by Tanzi (1978).

[8] A relationship of the form $A=a_0y$ could have been used at the cost of increasing the complexity of computation.

where σ is the depreciation rate of the capital stock and $\dot{k}$ is net investment, it can be seen that a combination of extreme credit rationing and high interest rates in the international markets could make it impossible for the economy to grow if $\dot{k}<(x-m_o/m_1)\ y-f/m_1-\sigma k$, implies $I<\sigma k$.

No further reference to the savings gap, i.e.

$$(11) \qquad I \leq (M-X) + (A-g) + (y-c-A),$$

will be made here since we will assume that consumption is centrally planned.

4. The solution of a linear control problem for an economy under a fiscal constraint.

In this section, we will discuss a simple linear optimal growth model that incorporates both the constraints originating from the financial side of the economy and those stemming from the previously discussed lack of responsiveness of some variables to changes in relative prices.

Therefore, it will be assumed that f and A are constant and that L=0. This implies that the fiscal constraint is binding and accordingly the proportion of government per capita expenditures will be exogenously set at $g^*=A-f$. This, in turn, means that the positive effect of public expenditure on GDP will also be given i.e.:

$$F(\epsilon g^*) = F[\epsilon(A-f)].$$

Considering u as a linear function and taking into account (3), (4), (6), (7) and (10), we have:

$$u = \{\Gamma(k)\ F[\epsilon(A-f)]\ (m_0-x+1)-g-(1-m_1)(\dot{k}+\sigma k)\}\ u_0, \tag{12}$$

The optimal path is obtained by maximizing

$$\int_0^T u\ e^{-\delta t}\ dt$$

where the variable $\dot{k}$ is subject to two constraints:

$$\dot{k} \leq [(x-m_0)/m_1]\ \Gamma(k)\ F[\epsilon(A-f)] - f/m_1 - \sigma k, \tag{13}$$

$$-\sigma k \leq \dot{k}. \tag{14}$$

The first one represents the external gap. The second one corresponds to the assumption that capital stock can only be reduced at the rhythm determined by the rate of depreciation: consumption must be smaller than total production because we are assuming that the capital stock cannot be consumed.

A typical solution to this problem is illustrated in Figure 1. The optimal path is composed of a combination of bang-bang and singular controls. The optimal capital stock on the singular path (k^*) is computed with the modified golden rule for this economy (the Euler equation):

$$\Gamma'(k^*) = [(\delta+\sigma)(1-m_1)]/F[\epsilon(A-f)]\ (1+m_0-x). \tag{15}$$

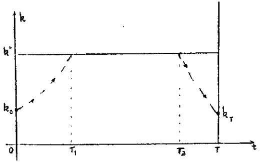

FIGURE 1

On the singular path, the marginal product of capital is directly proportional to the sum of both the time preference and the depreciation rates. The proportionality factor, in turn, is a parameter that depends positively on the imports GDP ratio and negatively on the exports one.

For this economy, it is optimal to stay near k^* as long as possible, so when the economy reaches k^*, $\dot{k}$ assumes a value equal to zero. On the other hand, if the economy starts at k_0 with $k_0 \neq k^*$, it would be optimal to "jump" from k_0 to k^* as quickly as possible. If $k_T \neq k^*$, the economy would stay at k^* until the "last minute" preceding T, and then jump to k_T. However, since the control variable is bounded by the constraints imposed by both the external gap (13) and the maximal depreciation rate (14), it cannot jump in any direction. Therefore, when $k_t = k^*$ it follows that either (13) or (14) are satisfied as an equality.

Hence, in the example depicted in Figure 1 -where $k_0 < k^*$ and

$k_T < k^*$ – the rate of growth is governed by (13) during the first stage –the external gap is binding–, the second stage is the modified golden rule solution obtained from (15) and, finally, the stock of capital is reduced at the highest feasible rate throughout the third stage.

This relationship between net investement $\dot{k}$ and the capital stock k is shown in Figure 2[9]. Starting at point A, on the schedule representing the external constraint, the economy moves from A to B where the optimal stock of capital k^* is reached. At B, there is a discontinuity in the $\dot{k}$ function that jumps from B to C. The economy remains at C (where $\dot{k}=0$) as long as posible and at the required time, the $\dot{k}$ function again jumps from C to D on the linear function representing the highest depreciation that can be achieved. The economy moves then from D to E, to end the programming period with a stock of capital equal to k_T.

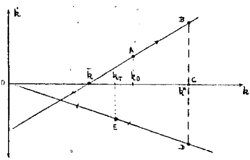

FIGURE 2

[9] Notice that we are working here with a linear approximation of (13) around $k=\bar{k}$.

We are now prepared to do some exercises aimed at getting further insight into the economic mechanisms that, after the debt crisis, produced the stylized facts that were already mentioned. Specifically, we want to illustrate the interaction between the fiscal and external gaps and the long-run capital stock under extreme credit rationing.

One of the most important consequences of the debt crisis was a sudden fall in capital inflows. In fact, most Latin-American countries became capital exporters. Accordingly, let's begin by assuming that f is unexpectedly increased. This will lower credit availability to the public sector, thereby reducing the provision of public goods and hence the right hand side of (15) will augment. The level of k^* at the stationary state will have to be reduced since $\Gamma''>0$:

$$dk^*/df = \Gamma'F'\epsilon/\Gamma''F < 0.$$

Similarly, given that:

$$dc^*/dk^* = \Gamma'F(1-x+m_0) - \sigma(1-m_1) = \delta(1-m_1) > 0;$$

a similar result will be obtained for consumption. The path that will be followed by k is depicted in Figure 3.

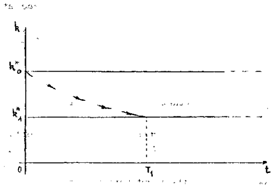

FIGURE 3

Following the reduction in foreign financing, the rate of growth is governed by:

$$\dot{k}/k = -\sigma$$

from the initial moment until it reaches the new stationary state at T_1. Under such conditions, the stock of capital would simply be left to evaporate at rate σ.

There is one point that should be stressed here. Notice that if $\epsilon=0$, the change in f would not modify the level of the capital stock k^*. That is, the impact effect of external credit rationing is stronger when production is influenced by the provision of public goods. And the existence of a fiscal/financial gap implies, precisely, that g must decrease if the government cannot finance its deficit domestically (i.e. if L=0). This is so because the dollarization of the economy impedes the government from substituting domestic public debt for external credit when facing an increasing f.

Since:

$$dk^*/dA = -\Gamma'F'\epsilon/\Gamma''F > 0$$

it is obvious that the government could offset the increment in f by increasing the tax burden. However, it proved to be very difficult to put such a policy into practice in the unstable macroeconomic environment of the eighties.

If the economy is in a situation where the external constraint (13) is binding when the increase in f takes place, the effect will be to restrain growth by reducing the velocity at which k^* could be reached. In fact from (13) it follows that for any given k:

$$d\dot{k}/df = -[1 + (x-m_0)\ \Gamma\ (k)\ F'\epsilon]/m_1 < 0.$$

Moreover, if the increase in f is large enough, the external constraint cannnot be met under any circumstance. This would happen if after the shock $\dot{k}=[(x-m_0/m_1)]\ \Gamma(k)F-(f/m_1)<-\delta k$. To meet the external constraint, net investment should be so negative that it would be impossible to depreciate the physical stock of capital at that rate. If this happened, the country would not fulfill the external commitments and would incur in arrears. This can be seen in Figure 4.

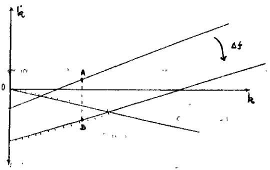

FIGURE 4

When f increases, k decreases from A to B on the external constraint. But B is not feasible, since in B: $\dot{k}<-\delta k$. If this happens, the set of controls is empty and the planning exercise is not possible.

The consequences of an increasing f in our model roughly conform with the observed facts after the debt crisis: the financial crack of the government and a huge fall in both investment and consumption. Likewise, it should be noticed that many Latin American countries incurred in arrears and showed investment levels that were not high enough to cover the depreciation of the capital stock. Therefore, given that the strategy of lowering investment to meet the external constraint is inconsistent, it is not surprising that the commercial banks (in an effort to recover at least a part of the loans) had proposed mechanisms aimed at exchanging debt for physical stocks of capital by means of debt equity-swap schemes[10].

5. Macroeconomic instability and economic policy: some perversities of disequilibrium.

The previous section showed the disturbing consequences that external credit rationing may induce on both the structure and the growth path of an economy. In this section, we discuss some of the consequences for economic policy evaluation that follow from our model. This issue deserves attention because policies that work under "normal" circumstances may have very distortive outcomes when applied in an economy undergoing deep disequilibria of the kind shown.

The first point regarding economic policy has to do with the size of the external financing needed to overcome the debt crisis and the feasible degree of fiscal adjustment that can be put into practice without inducing a worsening of the present disequilibria.

As was explained, if the external constraint is binding and the increase in f is strong, the rate of growth may not be big enough to reach the long-run optimal capital stock. In fact, this will occur whenever the external constraint implies $\dot{k}<0$. Were this the case, the economy would not be able to increase its capital stock an the dynamics would be ruled by (13) or (14) with a persistently falling capital stock.

It is apparent that under such conditions, given the external constraint, the capital stock is initially so low that the economy

[10] Several macroeconomic models have been constructed taking into account these solutions particularly referred to Argentina. Most of this exercises are simulations of the performance of the economy regarding some fundamental indicators.

These simulations -which use parameters and elasticities taken from empirical studies- are justified since the development policies currently proposed to deal with the crisis involve major changes in economic variables. Moreover, they are true policiy packages that encompass simultaneous changes in a large number of instruments, to the extent of rendering invalid the traditional comparative dynamics studies -see Frenkel and Rozenwurcel (1990), Fanelli and Chisari (1989), Fanelli and Frenkel (1989).

is not feasible in the long run and its capital stock will eventually disappear[11]. In drawing Figure 4 above, we assumed that this situation was due to a sudden increase in f. However, it should be noticed that a fiscal adjustment achieved by means of a huge fall in g would have similar consequences. The fall in g, by affecting the productive capacity of the economy might shift the external constraint downwards (as in the case of an increase in f) setting the economy on a negative growth path. This introduces a note of caution on policies oriented to stabilizing the economy by lowering public expenditure at any cost. In fact, this will most likely be the case in the long run if there is a huge fall in public investment.

The afore-mentioned characteristics of an economy under severe external credit rationing have another strong policy implication: if the economy showed a "too small" stock of capital, a "big push" of external financing would be necessary in order to overcome the inherent unstable character of the growth path.

The second point we want to stress here is that if the magnitude of the disequilibrium is as large as that represented in Figure 4, a kind of "hysteresis effect" can be observed. This means that history "matters" and that the consequences of both exogenous shocks and economic policy actions under such circumstances may not be strictly reversible. This fact adds further constraints to economic policy making.

To see how a "hysteresis effect" of shocks appears in the context of external credit rationing, assume, for the sake of the argument, that the initial capital stock is k_0 and that this

[11] Assume $\sigma=0$ and δ small enough. The stationary point corresponding to the solution of the Euler condition would be the capital level that determines a bliss point regarding production (perhaps infinite). However, $\bar{k}$ would be obtained solving $\Gamma(k) = f/(x-m_0)F$. If f is small, so is $\bar{k}$. Notice also that $d\bar{k}/dk = (x-m_0) F\Gamma'>0$ if Γ is monotonic.

economy is growing at a rate determined by the curve labeled I-I in Figure 5.

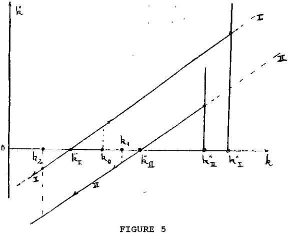

FIGURE 5

Let's consider that a sudden drop of external credit pushes the curve downwards to II-II when the economy has already arrived at k_1. If this change is significant, k_2 will be approximated; if the shock were reversed and the curve returned to I-I the economy might continue to persistently reduce its capital stock[12].

An analogous effect arises for natural resources under "depensation"[13]. History is relevant in the sense that reverting the decision with a delay may not lead the system to its initial

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[12] Recall that the stationary state $\bar{k}$ is determined by

$[(x-m_0)/m_1]\ \Gamma(\bar{k})\ F[\epsilon(A-f)] - f/m_1 - \sigma\bar{k} = 0.$

However, it is unstable, and so comparative static exercises would have only a reference character relative to the position of curves like I-I or II-II.

[13] See Clark (1976).

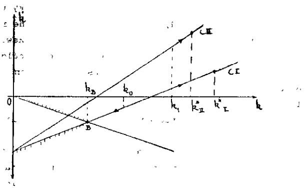

FIGURE 7

If the economy is initially at k_0, the capital stock will persistently decrease at the rhythm determined by CI (the external constraint). When arriving at B, the differential equation governing the change of k is indeterminate. A contradiction arises between the admissible maximum rate of depreciation, and the necessary minimum rate of growth to fulfill the external gap. In fact, the set of feasible policies is empty[14].

Assume instead that the initial condition is given by k_1 in Figure 7. The economy can grow until it reaches k^*_I; this bliss capital stock will be possible only if the initial capital stock is large enough.

Therefore an increase in x (or a reduction of import coefficients) will have two effects. On the one hand, it will

[14] If the economy is extremely poor, i.e., if the initial capital stock is below the threshold given by k_B for example, then no policy will be feasible and the only way out will be to make endogenous some (given) parameter (for example f).

decrease the long-run capital stock (from k^*_{I} to k^*_{II}) and consumption; it will not be an interesting policy to follow if the capital endowment is relatively high. On the other hand, it helps to make the economy feasible; if the capital endowment is relatively low at the initial moment (like k_0), that policy will allow the economy to increase k at the rate of growth given by CII, though converging at a more modest level of consumption at the stationary state.

6. Concluding remarks.

We have here presented the three-gap model in an optimal growth framework by replacing the traditional isoperimetric condition for a financial constraint that has to be fulfilled at any moment.

We have also presented the "crowding-in" effect that appears in the three-gap literature as the result of the existence of public goods or public factors of production, that could not be provided by private agents and that are critical for the possibilities of production of the whole economy.

We have shown that the impact effect of external credit rationing could be very strong when production is influenced by the provision of public goods, modifying not only the dynamics of the economy but also the long-run capital stock. Moreover, we have illustrated how exogenous shocks or policy actions could not be time-reversible and could lead the economy up a blind alley. A "hysteresis" effect could arise under these conditions: reverting the decision with some delay could not help the economy to return to the initial situation.

The model was also used to evaluate the impact of "export-led" and "import-substitution" strategies on long-run capital and on the dynamics of the economy.

However, it should be mentioned that important issues concerning economic policy have not been analyzed here.

First of all, it has to be emphasized that we have not been dealing with the savings gap, traditionally included as one of the binding constraints. One important consequence of this simplification is that problems regarding income distribution have been eliminated. This issue is clearly related to the postkeynesian solution of the razor's edge problem in Harrod-Domar's model. To sustain a high rate of growth, a consistent level of savings should be maintained; however the major savers in the economy are non-wage earners and, henceforth, income distribution should be biased in their favor.

A second important aspect refers to the hypothesis of persistent intertemporal equilibrium of factor markets. Implicitly, that assumption was actually adopted when defining all variables in per capita terms. The weakness of this procedure comes from the fact that when gaps exist the so-called labor-gap might arise.[15]

Finally, the role of the financial crisis has been minimized except for the fact that all the analysis is conducted by initially replacing the isoperimetric constraints by instantaneous constraints, which really means that international credit markets are closed to debtors. The costs imposed by government inflationary financing and the dangerous hyperinflationary episodes -actually observed- have not been discussed.

[15] See Fanelli and Chisari (1989) for the discussion of the income-distribution and the labor-gaps.

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